CORRELATIONS IN PARTICLE PRODUCTION AND SQUEEZING PHENOMENA

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Abstract

Recent developments in studies of multiparticle correlations in high energy particle collisions are reviewed. Both experimental data and theoretical results in quantum chromodynamics are discussed. Application of the developed methods to the coherent, squeezed and correlated states of photons is considered. Some speculations concerning possible coherent and squeezed states of pion fields as well as their specific features are described.

1. INTRODUCTION

The study of correlations of quantum fields is a common subject in statistical physics, quantum optics, multiparticle production. Therefore, it is not at all surprising to find out common methods of research in those fields. I shall describe some newly obtained results in studies of correlations of particles produced in high energy collisions as well as their relation to the correlations of photons and, in particular, to the squeezing phenomenon.

The nature of any source of radiation (of photons, gluons or other entities) can be studied by analyzing multiplicity distributions, energy spectra, various correlation properties, etc. A particular example is provided by the coherent states of fields which give rise to Poisson distribution. However, in most cases one has to deal with various distributions revealing different dynamics. In particle physics dealing with strong interactions, one relies on quantum chromodynamics (QCD) as a substitute of quantum electrodynamics (QED) in case of photons. The fields of quarks and gluons are considered instead of electrons and photons. Let us stress that quarks are described by spinors similar to electrons, and gluons are massless vector particles like photons. The important difference is that gluons carry the color charge while the

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photons are electrically neutral. It gives rise to the self-interaction of gluons as well as to famous properties of the asymptotic freedom and confinement.

Let us turn now to some experimental facts about high energy particle interactions. When two high energy particles collide, the bunch of new particles is produced in each event. It is common to plot the distribution of those events in the number of particles produced (the multiplicity distribution, for short). Usually, they differ drastically from the Poisson distribution, clearly indicating strong correlations. Several years ago the experimentalists of UA5 Collaboration in CERN noticed [1] a shoulder in the multiplicity distribution of particles produced in $p\bar{p}$ collisions at energies ranging from 200 to 900 GeV in the center of mass system. (Let me remind here that the mass of the proton is about 1 GeV, i.e. much smaller.) It looked like a small wiggle over a smooth curve and was immediately ascribed by theorists to processes with larger number of Pomerons exchanged in the traditional schemes. More recently, several collaborations studying e^+e^- collisions at 91 GeV in CERN reported (see, e.g., [2, 3]) that they failed to fit the multiplicity distributions of produced particles by smooth curves (the Poisson and Negative Binomial distributions were among them). Moreover, subtracting such smooth curves from the experimental ones they found steady oscillatory behaviour of the difference. It was ascribed to the processes with different number of jets.

The new sensitive method of theoretical analysis of distributions was proposed in [4, 5] (for the review see [6]). It appeared as a byproduct of the solution of the equations for generating functions of multiplicity distributions in quantum chromodynamics (QCD). It appeals to the moments of the multiplicity distribution. According to QCD, the so–called cumulant moments (or just cumulants), described in more details below, should reveal the oscillations as functions of their ranks, while they are identically equal to zero for the Poisson distribution and are steadily decreasing positive functions for Negative Binomial distribution so widely used in phenomenological analysis. Experimental data show the oscillatory behaviour of cumulants (see [7]) of multiplicity distributions in high energy inelastic processes initiated by various particles and nuclei, even though some care should be taken due to the high multiplicity cut-off of the data. When applied to the squeezed states, the method demonstrates [8] the oscillations of cumulants in slightly squeezed states and, therefore, can be useful for their detection. In such circumstances one is tempted to speculate about the alternative explanation when considering possible similarity of these findings to typical features of squeezed and correlated states. However, first let us describe briefly the

method of analysis of the distributions which we rely upon.

2. DISTRIBUTION FUNCTION AND ITS MOMENTS.

Here I summarize definitions and notations for the values that are used to characterize various processes of inelastic scattering according to number of particles produced in these processes. The relations connecting these values to each other and the examples related to some distributions typical in the probability theory are provided.

Any process of inelastic scattering (that is the scattering with new particles produced) can be characterized by the function P_n, the multiplicity distribution function. The value of P_n denotes the probability to observe n particles produced in the collision. It is clear that P_n must be normalized to unity:

$$\sum_{n=0}^{\infty} P_n = 1. \tag{2.1}$$

Sometimes the multiplicity distribution of particles produced can be conveniently described by its moments. It means that the series of numbers P_n is replaced by another series of numbers according to a certain rule. All these moments can be obtained by the differentiation of the so–called generating function $G(z)$ defined by the formula:

$$G(z) = \sum_{n=0}^{\infty} P_n z^n. \tag{2.2}$$

Thus, instead of the discrete set of numbers P_n we can study the analytical function $G(z)$.

We will use the factorial moments and cumulants defined by the following relations:

$$F_q = \frac{\sum_{n=0}^{\infty} P_n(n-1)\cdots(n-q+1)}{(\sum_{n=0}^{\infty} P_n n)^q} = \frac{1}{\langle n \rangle^q} \frac{d^q G(z)}{dz^q}\bigg|_{z=1}, \tag{2.3}$$

$$K_q = \frac{1}{\langle n \rangle^q} \frac{d^q \ln G(z)}{dz^q}\bigg|_{z=1}, \tag{2.4}$$

where

$$\langle n \rangle = \sum_{n=0}^{\infty} P_n n \tag{2.5}$$

is the average multiplicity.

Reciprocal formulas expressing the generating function in terms of cumulants and factorial moments can also be obtained:

$$G(z) = \sum_{q=0}^{\infty} \frac{z^q}{q!} \langle n \rangle^q F_q$$
$$(F_0 = F_1 = 1), \tag{2.6}$$

$$\ln G(z) = \sum_{q=1}^{\infty} \frac{z^q}{q!} \langle n \rangle^q K_q$$
$$(K_1 = 1). \tag{2.7}$$

The probability distribution function itself is related to the generating function in the following way:

$$P_n = \frac{1}{n!} \frac{d^n G(z)}{dz^n} \bigg|_{z=0}. \tag{2.8}$$

Factorial moments and cumulants are connected to each other by the following recursion relation:

$$F_q = \sum_{m=0}^{q-1} C_{q-1}^m K_{q-m} F_m, \tag{2.9}$$

where

$$C_{q-1}^m = \frac{(q-1)!}{m!(q-m-1)!}$$

are binomial coefficients. Relation (2.9) gives the opportunity to find the factorial moments if cumulants are known, and vice versa.

It must be pointed out that cumulants are very sensitive to small variations of the distribution function and hence can be used to distinguish the distributions which otherwise look quite similar.

Usually, cumulants and factorial moments for the distributions occuring in the particle physics are very fast growing with increase of their rank. Therefore sometimes it is more convenient to use their ratio:

$$H_q = \frac{K_q}{F_q}, \tag{2.10}$$

which behaves more quietly with increase of the rank q remaining a sensitive measure of the tiny details of the distributions.

In what follows we imply that the rank of the distribution function moment is non–negative integer even though formulas (2.3) and (2.4) can be generalized to the non–integer ranks.

Let us demonstrate two typical examples of the distribution.

1. *Poisson   distribution.*

The Poisson distribution has the form:

$$P_n = \frac{\langle n \rangle^n}{n!} \exp\left(-\langle n \rangle\right). \tag{2.11}$$

Generating function (2.2) can be easily calculated:

$$G(z) = \exp\left(\langle n \rangle z\right). \tag{2.12}$$

According to (2.3) and (2.4) we have for the moments of this distribution:

$$F_q = 1, \quad K_q = H_q = \delta_{q1}. \tag{2.13}$$

2. *Negative   binomial   distribution.*

This distribution is rather successfully used for fits of main features of experimental data in particle physics. It has the form:

$$P_n = \frac{\Gamma(n + k)}{\Gamma(n + 1)\Gamma(k)} \left(\frac{\langle n \rangle}{k}\right)^n \left(1 + \frac{\langle n \rangle}{k}\right)^{-(n+k)}, \tag{2.14}$$

where Γ is the gamma function, and k is a fitting parameter.

At $k = 1$ we have the usual Bose distribution. The Poisson distribution can be obtained from (2.15) in the limit $k \to \infty$.

Generating function for the negative binomial distribution reads:

$$G(z) = \left(1 - \frac{z \langle n \rangle}{k}\right)^{-k}, \tag{2.15}$$

and the moments of this distribution are:

$$F_q = \frac{\Gamma(k + q)}{\Gamma(k) k^q},$$
$$K_q = \frac{\Gamma(q)}{k^{q-1}},$$
$$H_q = \frac{\Gamma(q)\Gamma(k + 1)}{\Gamma(k + q)}. \tag{2.16}$$

3. QCD EQUATIONS AND MOMENTS OF DISTRIBUTIONS.

The multiparticle production processes are described in quantum chromodynamics as a result of the interaction of quarks and gluons which leads to creation of additional quarks and gluons forming the observed hadrons at the very last stage. The most typical features of the processes are determined by the vector nature of gluons and by the dimensionless coupling constant. The gluons are colour charged in distinction to photons which have no electric charge. Therefore, they can emit gluons in addition to quark-antiquark pairs. That is why both quark and gluon jets are considered in quantum chromodynamics as main objectives. Their development is described by the evolution equations. The main parameter of the evolution is the opening angle of the jet or its transverse momentum. The subsequent emission of gluons and quarks fills in the internal regions of the previously developed angular cones so that they do not overlap (angular ordering). This remarkable property can be exploited to formulate the probabilistic scheme for the development of the jet as a whole. Then its evolution equations remind the well-known classical Markovian equations for the "birth–death" (or "mother–daughter") processes. (The detailed discussion of that approach, based on the coherence phenomenon, see in [9]).

It is quite natural to start our studies with the simplest case of gluodynamics. There are no quarks in that case, and interactions of gluons are considered only. The system of equations degenerates to the single equation

$$G'(y) = \int_0^1 dx K(x) \gamma_0^2 [G(y + \ln x) G(y + \ln(1 - x)) - G(y)], \qquad (3.1)$$

where $G'(y) = dG/dy$, y is the evolution parameter,

$$\gamma_0^2 = \frac{6\alpha_S}{\pi}, \qquad (3.2)$$

α_S replaces α of electrodynamics and the kernel of the equation is

$$K(x) = \frac{1}{x} - (1 - x)[2 - x(1 - x)]. \qquad (3.3)$$

It is the non-linear integro-differential equation with shifted arguments in the non-linear term which take into account the energy conservation. It can be reduced to the equation for the moments [5] and solved. The most prominent feature of the solution are oscillations of cumulants (or ratio H_q)

; see [6]. It has been confirmed by experiment as shown shown in fig. 1. It differs from all previously considered phenomenological distributions and, from the mathematical point of view, is interesting since it implies that QCD deals with non-infinitely divisible distributions in contradistinction to conventional ones. In particular, it forbids Poissonian cluster models so popular in physics modelling.

It is interesting to note that it corresponds to rather smooth distribution, i.e. there is no obvious one-to-one correspondence between the shapes of distributions and behaviour of their cumulants.

4. APPLICATION TO PHOTONS.

The above methods can easily be applied to photons [8]. Since the generating functions of photon distributions for the squeezed and correlated states are known [10] one can use the equations of section 2 and get all the required moments. Let us consider the most general mixed squeezed state of one–mode light. The generating function of the photon number distribution was obtained in [11]:

$$ G(u) = P_0 \left[\left(1 - \frac{u}{\lambda_1} \right) \left(1 - \frac{u}{\lambda_2} \right) \right]^{-1/2} \exp \left[\frac{u\xi_1}{u - \lambda_1} + \frac{u\xi_2}{u - \lambda_2} \right] , \qquad (4.1) $$

$$ \lambda_1 = \left(\sqrt{R_{11}R_{22}} - R_{12} \right)^{-1} , \qquad \lambda_2 = - \left(\sqrt{R_{11}R_{22}} + R_{12} \right)^{-1} , $$

$$ \xi_1 = \frac{1}{4} \left(1 - \frac{R_{12}}{\sqrt{R_{11}R_{22}}} \right) \left(y_1^2 R_{11} + y_2^2 R_{22} - 2\sqrt{R_{11}R_{22}}y_1y_2 \right) , $$

$$ \xi_2 = \frac{1}{4} \left(1 + \frac{R_{12}}{\sqrt{R_{11}R_{22}}} \right) \left(y_1^2 R_{11} + y_2^2 R_{22} + 2\sqrt{R_{11}R_{22}}y_1y_2 \right) , $$

$$ R_{11} = \left(T + 2d + \frac{1}{2} \right)^{-1} (\sigma_{pp} - \sigma_{qq} - 2i\sigma_{pq}) = R_{22}^*, $$

$$ R_{12} = \left(T + 2d + \frac{1}{2} \right)^{-1} \left(\frac{1}{2} - 2d \right), \qquad\qquad (4.2) $$

and

$$ y_1 = y_2{}^* = \left(T - 2d - \frac{1}{2} \right)^{-1} [(T - 1)\langle z^* \rangle + (\sigma_{pp} - \sigma_{qq} + 2i\sigma_{pq})\langle z \rangle] . \quad (4.3) $$

The complex parameter $\langle z \rangle$ is given by relation

$$\langle z \rangle = 2^{-\frac{1}{2}} \left(\langle q \rangle + i \langle p \rangle \right). \tag{4.4}$$

$$
\begin{aligned}
m_{11} &= \sigma_{pp} = \mathrm{Tr}\left(\hat{\varrho}\hat{p}^2 \right) - \langle p \rangle^2, \\
m_{22} &= \sigma_{qq} = \mathrm{Tr}\left(\hat{\varrho}\hat{q}^2 \right) - \langle q \rangle^2, \\
m_{12} &= \sigma_{pq} = \frac{1}{2}\mathrm{Tr}\ \left[\hat{\varrho}(\hat{p}\hat{q} + \hat{q}\hat{p}) \right] - \langle p \rangle \langle q \rangle,
\end{aligned} \tag{4.5}
$$

$$T = \mathrm{Tr}\ \mathbf{m} = \sigma_{pp} + \sigma_{qq}$$

,

$$d = \det \mathbf{m} = \sigma_{pp}\sigma_{qq} - \sigma_{pq}^2$$

, and σ_{ij} are the well known elements of the dispersion matrix.

The photon distribution function exhibits an oscillatory behaviour if we deal with highly squeezed states ($T = \sigma_{pp} + \sigma_{xx} \gg 1$) for large values of the parameter z. A question arises: is it possible to obtain a similar "abnormal" behaviour of other characteristics of the photon distribution, namely, introduced in section 2 cumulants, factorial moments and their ratio H_q ? If yes, then in what region of parameters can such anomalies reveal themselves?

The direct differentiation of function $\ln G(u)$ at $u = 1$ yields the cumulants (see section 2)

$$K_q = \frac{(q-1)!}{\langle n \rangle^q} \left[\frac{1}{(\lambda_1 - 1)^q} \left(\frac{1}{2} + q\frac{\xi_1 \lambda_1}{1 - \lambda_1} \right) + \frac{1}{(\lambda_2 - 1)^q} \left(\frac{1}{2} + q\frac{\xi_2 \lambda_2}{1 - \lambda_2} \right) \right], \tag{4.6}$$

with the average number of photons $\langle n \rangle$ [11]

$$\langle n \rangle = \frac{T - 1}{2} + |z|^2 \ .$$

Now let us consider the case of the slightly squeezed state, $y = (T - 1) \ll 1$ when the photon distribution function does not oscillate. Impose also an additional condition

$$\gamma = \frac{|z|^2}{\sqrt{y/2}} \gg 1 \ ,$$

that makes possible to obtain approximate formulas for the functions K_q, F_q, and H_q. For K_q, we have the following approximate expression:

$$K_q = q!(-1)^{q-1}\gamma^{1-q} \ . \tag{4.7}$$

Then recursion relation (2.9) yields:

$$F_q = q!(-1)^q\gamma^{-q}L_q^{-1}(\gamma) \ , \tag{4.8}$$

where $L_q^{-1}(x)$ are generalized Laguerre polynomials. For H_q, with $q \ll \gamma$ we have:

$$H_q = K_q/F_q = -\frac{\gamma}{L_q^{-1}(\gamma)} \approx (-1)^{q+1}q!\gamma^{1-q} \ll 1 \ . \tag{4.9}$$

(If $\gamma \gg q$ the term with highest power of γ dominates over the rest of sum in $L_q^{-1}(\gamma)$, and $F_q \to 1$ as for Poisson distribution). The exact shape of the function H_q is shown in fig.2^a. The distribution function P_n does not oscillate (fig.2^b).

However, the most abrupt oscillations of the functions K_q and H_q have been obtained when $(T-1) \ll 1$, but condition $\gamma \gg 1$ is not valid. The corresponding curves are shown in figs.3^a, 3^b. Note that the photon distribution function is smooth again being approximately equal to zero at $q \neq 1$.

The most regular oscillating patterns of K_q and H_q are seen at $(T-1) \sim 0.1$, $|z| \sim 1$ (figs.4^a, 4^b). Finally we consider the opposite case when the photon distribution function P_n exhibits strong oscillations while K_q and H_q behave smoothly. Such a behaviour is typical at $T \sim 100$, $|z| \sim 1$ when K_q exponentially grows while H_q monotonically decreases with q (fig.5).

Thus we have shown that the cumulants of the photon distribution function for one–mode squeezed and correlated light at finite temperature possess strongly oscillating behaviour in the region of slight squeezing where the photon distribution function itself has no oscillations. And vice versa in the region of large squeezing, where the photon distribution function strongly oscillates, the cumulants behave smoothly. Hence, the behaviour of cumulants may provide a very sensitive method of detecting very small squeezing and correlation phenomena due to the presence of strong oscillations.

Let us note also that these methods were successfully applied to predicting some regularities in behaviour of lasers in the lasing regime [12].

5. COHERENT AND SQUEEZED STATES IN PION PHYSICS.

The notion of coherent and squeezed states is not widely applied in pion production yet even though several papers deal with the subject [13], [14],[15],[16],[17],[18],[19],[20],[21],[22]. The most striking difference with photon case appears due to the requirement of isotopic spin conservation inherent in pion physics but absent for photons. It gives rise to strong charge asymmetry of produced events. If the isospin zero projection of the coherent state is considered, it predicts many events without neutral pions, for example. It is demonstrated in fig. 6 borrowed from [15]. It used to be related to the so-called Centauro-type events observed in cosmic rays.

Let us consider for definitiveness the process of the collision of two high energy nucleons, where in the final state, apart from the two nucleons, a certain number of pions has been created. The total isotopic spin of the pion system is limited, according to the conservation laws, to the values $I = 0, 1, 2$, whereas in the general case it could take values up to n_{tot}, where n_{tot} is a total number of pions. This fact significantly affects the pion charge distribution. Even if the distribution in the total pion number n_{tot} is poissonian (as it happens when pions are produced by classical currents), it turns out, that the separate distributions of the charged (n_+, n_-) or neutral (n_0) pions are much wider than the Poisson one (see fig.6). The experimental confirmation of this fact could be the "Centauro" events, where the charged particles are noticeably dominating, or "anti-Centauro" ones with a large number of neutral pions.

Let us illustrate the idea by considering, following the quasiclassical approach of refs.[15],[18], the production of many pions in a system with zero isotopic spin. The characteristic initial assumption is a possibility of describing the pion system that radiates the final state pions as a classical field , i.e. that the number of pions per phase space cell is assumed to be big. According to the standard reduction formula, the amplitude of generation of N pions by the source J equals

$$A^{a_1,\ldots,a_n}(k_1,\ldots,k_n) = \lim_{k_n^2 \to m_\pi^2} \int D\pi^a \int DJ^a W[J]$$

$$\exp(iS[\pi] + i \int d^4x \pi^a J^a) \prod_{n=1}^{N} \int d^4x_n e^{ik_n x_n}(-\partial_{x_n}^2 - m_\pi^2)\pi^{a_n}(x_n), \quad (5.1)$$

where the functional integration over J corresponds to the averaging over the characteristics of the pion source. Let us notice, that the radiation of

a classical current exactly reproduces the language of coherent states. The specific feature of pion fields is that one projects the coherent states onto the states with a definite isotopic spin, and it gives rise to drastical change of final charge distributions. The quasiclassical estimate of the amplitude, performed in the assumption on the axial symmetry of the initial interaction and on the isotopic symmetry of the pion system (i.e. of the zero total isospin) [14],[15],[18], leads to the two characteristic conclusions that also appear in other publications on this topic.

Firstly, only the distribution over the total number of pions is poissonian. The distributions over the number of neutral and charged pions are much wider (see fig. 6). For the state with zero isospin a probability of finding $2n$ neutral pions in the system of $2N$ pions has a form [13],[19]:

$$P(n, N) = \frac{(N!)^2 2^{2n}(2n)!}{(n!)^2 2^{2N}(2N+1)!}.$$ (5.2)

At large n, N we have a characteristic distribution [23]:

$$P(n, N) \sim (n/N)^{-1/2}.$$ (5.3)

Secondly, the conservation of the total isospin leads, for example, to the specific angular correlations between the particles with different charges. Let us give the characteristic formula for the correlation over the azimuthal angle φ, obtained in [24] for the pions having zero rapidity:

$$\frac{(\sigma_{tot}\frac{d\sigma^{\pi^+\pi^-}}{dk_1 dk_2} - \frac{9}{10}\frac{d\sigma^{\pi^+}}{dk_1}\frac{d\sigma^{\pi^-}}{dk_2})}{(\frac{d\sigma^{\pi^+}}{dk_1}\frac{d\sigma^{\pi^-}}{dk_2})} = \frac{3}{10}\cos^2(\varphi_1 - \varphi_2).$$ (5.4)

The experimental verification of such predictions is to our opinion very interesting.

The similar procedure of projecting *squeezed* states onto the states with definite isospin has been attempted in [25]. It produced some interesting modifications of fig. 6 which we have no space to discuss here.

In the recent literature a "disoriented chiral condensate" was widely discussed as a possible asymmetry source in the production of charged and neutral pions in some fraction of the events (in particular, in the above-mentioned "Centauros"). Let us remind, that the transformation properties of mesons with respect to the chiral transformations are determined according to the corresponding properties of the order parameter, which characterizes the spontaneous breaking of chiral symmetry and is given, for example,

by the average from the bilinear combination of quark fields

$$\Phi \sim \langle \bar{q}_L q_R \rangle, \tag{5.5}$$

where $q_{R(L)}$ are the right (left) states of the massless quarks. For investigating the character of the singularity of thermodynamical functions in the vicinity of the phase transitions, it is desirable to find a solvable model having the same symmetry. Then, according to the universality principle based on the scale invariance near the critical point, the solutions of this model will have the same set of singularities. In such an approach the order parameter (5.5) can be rewritten in terms of a set of hadron fields having the same symmetry. Therefore, the multipion states become related to the massless quark fields and quark-gluon plasma, i.e. there appears a hope to describe a phase transition between these so differing phases within such a model. In the realistic case of two massless quark flavors the chiral field can be written in the form

$$\Phi \sim \sigma \bullet \bar{1} + i\bar{\tau} \bullet \bar{\pi}, \tag{5.6}$$

where $\sigma, \bar{\pi}$ are the real fields, $\bar{\tau}$ are the standard Pauli matrices, the π - meson fields $\bar{\pi}$ form an isotriplet and σ is an isosinglet.

In the case of $SU(3)$ - algebra the number of such fields is already 18. They form the scalar and pseudoscalar nonets.

The dynamics of these degrees of freedom can be described by the lagrangian of the linear σ model

$$L = \frac{1}{2}(\partial \sigma)^2 + \frac{1}{2}(\partial \bar{\pi})^2 - V(\sigma, \bar{\pi}), \tag{5.7}$$

where V is a potential depending on the combination $\sigma^2 + \bar{\pi}^2$. In the standard version [26] the spontaneous symmetry breaking occurs via the formation of the nonzero vacuum average of the field σ. The isotriplet fields remain massless, i.e. the pions are the goldstones of the chiral group.

Let us now assume [17], that in some region of space the vacuum orientation is different from the standard one, and, for example,

$$\langle \sigma \rangle = f_\pi \cos \theta, \quad \langle \bar{\pi} \rangle = f_\pi \bar{n} \sin \theta, \tag{5.8}$$

where $f_\pi = 93$ MeV, and $\bar{n}$ is a unit orientation vector of $\bar{\pi}$. Such an assumption presupposes a specific scenario of the process, which is still not studied in details.

If the field Φ is isotropic with respect to a direction on the 3-dimensional sphere in the 4-dimensional space with the angles defined as

$$(\sigma, \pi_3, \pi_2, \pi_1) = (\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi\sin\eta, \sin\theta\sin\phi\cos\eta), \quad (5.9)$$

then for the probability distribution of a given state $r \equiv \cos^2\phi$ we have:

$$\int_{r_1}^{r_2} dr\, P(r) = \frac{1}{\pi^2} \int_0^{2\pi} d\eta \int_0^{\pi} d\theta \sin^2\theta \int_{\arccos r_2^{1/2}}^{\arccos r_1^{1/2}} d\phi \sin\phi, \quad (5.10)$$

as it was obtained for the first time in refs.[15, 23]:

$$P(r) = \frac{1}{2\sqrt{r}}, \quad (5.11)$$

i.e. we have again returned to the formula (5.2). Thus the probability of finding an event with a fraction of the neutral pions being smaller than a certain given r_0 is

$$W(r < r_0) = \sqrt{r_0}, \quad (5.12)$$

which constitutes 10 % even at $r_0 = 0.01$. The charge fluctuations in such a system are much larger than those that would follow from the Poisson distribution, where the distributions are concentrated around $r=1/3$.

Let us note that the model of creation of N isoscalar pion pairs is described as

$$|\Psi\rangle \propto (2a_+^+ a_-^+ - (a_0^+)^2)^N |0\rangle, \quad (5.13)$$

which is reminiscent of squeezed states projected onto zero isospin but is more restrictive than that projection. (Here $a_{+,-,0}^+$ are the creation operators of positive, negative, neutral pions, correspondingly).

6. CONCLUSIONS

We have shown that the correlation analysis can be done by similar methods for photons and pions. Some further understanding of the relation between distribution behaviour and features of its moments is awaited for. More clear insight into usefulness of the idea of coherent and squeezed states as applied to particle physics is required. However, the very first attempts in these directions described above are very encouraging.

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